

Charged Lepton Mass Relations in a Supersymmetric Yukawaon Model

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Abstract

According to an idea that effective Yukawa coupling constants Y_f^{eff} are given vacuum expectation values $\langle Y_f \rangle$ of fields (“Yukawaons”) Y_f as $Y_f^{eff} = y_f \langle Y_f \rangle / \Lambda$, a possible superpotential form in the charged lepton sector under a $U(3)$ [or $O(3)$] flavor symmetry is investigated. It is found that a specific form of the superpotential can lead to an empirical charged lepton mass relation without any adjustable parameters.

1 Introduction

The so-called “Yukawaon model” [for example, see Ref.[1]] claims that, in effective Yukawa interactions of quarks and leptons

$$H_Y = \sum_{i,j} \ell_i (Y_e^{eff})_{ij} e_j^c H_d + \cdots, \quad (1.1)$$

the effective Yukawa coupling constants Y_f^{eff} ($f = e, \nu, u, d$) are given by the vacuum expectation values (VEVs) $\langle Y_f \rangle$ of a scalar field Y_f as

$$Y_f^{eff} = \frac{y_f}{\Lambda} \langle Y_f \rangle. \quad (1.2)$$

Here, for simplicity, we have explicitly denoted only the charged lepton sector. In Eq.(1.1), ℓ and e^c are $SU(2)_L$ doublet and singlet fields, respectively, and Λ is an energy scale of the effective theory. (We have considered a supersymmetric (SUSY) scenario.) Hereafter, we refer the fields Y_f as “Yukawaons” [1], which are gauge singlets. In addition to the Yukawaon Y_e , we consider a field Φ_e which is related to Y_e as

$$\langle Y_e \rangle = k \langle \Phi_e \rangle \langle \Phi_e \rangle. \quad (1.3)$$

We also refer Φ_e as a “ur-Yukawaon”, which has been introduced in order to fix the VEVs of the Yukawaon Y_e . (For the moment, we consider the ur-Yukawaon only in the charged lepton sector.) Then, an empirical charged lepton mass formula [2]

$$R_e \equiv \frac{m_e + m_\mu + m_\tau}{(\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau})^2} = \frac{2}{3}, \quad (1.4)$$

is rewritten as

$$R_e \equiv \frac{v_1^2 + v_2^2 + v_3^2}{(v_1 + v_2 + v_3)^2} = \frac{2}{3}, \quad (1.5)$$

where $v_i = \langle (\Phi_e)_{ii} \rangle$.

Previously, the author [3] has derived the relation (1.5) by assuming the following U(3)-flavor-invariant scalar potential

$$V = \mu^2(\pi^2 + \eta^2 + \sigma^2) + \lambda(\pi^2 + \eta^2 + \sigma^2)^2 + \lambda'(\pi^2 + \eta^2)\sigma^2, \quad (1.6)$$

where

$$\pi = \frac{1}{\sqrt{2}}(\Phi_{11} - \Phi_{22}), \quad \eta = \frac{1}{\sqrt{6}}(\Phi_{11} + \Phi_{22} - 2\Phi_{33}), \quad \sigma = \frac{1}{\sqrt{3}}(\Phi_{11} + \Phi_{22} + \Phi_{33}), \quad (1.7)$$

and $\pi^2 + \eta^2 + \sigma^2$ and σ^2 correspond to $\text{Tr}[\Phi\Phi]$ and $\frac{1}{3}\text{Tr}^2[\Phi]$ in a diagonal basis of the VEV matrix $\langle \Phi \rangle$, respectively. Here, we have dropped the index “e” in Φ_e for convenience. Since, in the present paper, we often meet with traces of matrices A , hereafter, we denote the traces $\text{Tr}[A]$ as $[A]$ concisely. The scalar potential (1.6) can be rewritten as

$$V = \mu^2[\Phi\Phi] + \lambda[\Phi\Phi]^2 + \frac{1}{3}\lambda'[\Phi^{(8)}\Phi^{(8)}][\Phi]^2, \quad (1.8)$$

where $\Phi^{(8)}$ is an octet part of the nonet field Φ , $\Phi^{(8)} = \Phi - \frac{1}{3}[\Phi]$. The minimizing condition¹ of V demands

$$\frac{\partial V}{\partial \Phi} = 2 \left(\mu^2 + 2\lambda[\Phi\Phi] + \frac{1}{3}\lambda'[\Phi]^2 \right) \Phi + \frac{2}{3}\lambda' \left([\Phi\Phi] - \frac{2}{3}[\Phi]^2 \right) [\Phi] = 0, \quad (1.9)$$

so that we obtain the relation (1.5), i.e.

$$R = \frac{[\Phi\Phi]}{[\Phi]^2} = \frac{2}{3}, \quad (1.10)$$

together with $\mu^2 + 2\lambda[\Phi\Phi] + \frac{1}{3}\lambda'[\Phi]^2 = 0$.² Of course, a statement that the relation (1.10) was derived by assuming U(3) symmetry is not correct. The accurate statement is that the relation (1.10) was derived from a scalar potential (1.6) [(1.8)] which is invariant under U(3) symmetry, but which is not a general form of the U(3) invariant scalar potential.

A straightforward SUSY version of the scalar potential (1.8) is as follows: the superpotential W is given by

$$W = \mu[\Phi A] + \lambda'[\Phi][\Phi^{(8)}B], \quad (1.11)$$

where A and B are additional nonet fields. Then, the superpotential (1.11) leads to a scalar potential

$$V = |\mu|^2[\Phi\Phi^\dagger] + |\lambda'|^2[\Phi][\Phi]^\dagger[\Phi^{(8)}\Phi^{(8)\dagger}] + \dots. \quad (1.12)$$

¹ The stability condition $\partial^2 V / \partial \Phi \partial \Phi > 0$ will put a further constraint for a value of λ'/λ .

² The condition (1.9) has a form $c_1 \langle \Phi \rangle + c_0 \mathbf{1} = 0$, so that the eigenvalues v_i of $\langle \Phi \rangle$ are given by $v_i = -c_0/c_1$. Since we know that the observed charged lepton masses m_{ei} have nonzero and nondegenerate values, we want a set of (v_1, v_2, v_3) with nonzero and nondegenerate values. Then, we must require $c_1 = c_0 = 0$, so that we obtain the result (1.10).

However, although the minimizing condition of the scalar potential (1.12) can lead to the relation (1.10), the vacuum is not stable, because there is another lower vacuum $V = 0$ (a SUSY vacuum) at $\langle \Phi \rangle = 0$.

A supersymmetric approach with SUSY vacuum conditions to the mass relation (1.4) has first been done by Ma [4]. His model with a flavor symmetry $\Sigma(81)$ is impeccable, but somewhat intricate. Stimulated by his work, the author [5] has also proposed a superpotential with a simple form

$$W = \mu[\Phi\Phi] + \lambda[\Phi\Phi\Phi], \quad (1.13)$$

by assuming a Z_2 symmetry in addition to the $U(3)$ flavor symmetry. Here, it has been assumed that the octet $\Phi^{(8)} = \Phi - \frac{1}{3}[\Phi]$ and singlet $\Phi^{(1)} = \frac{1}{3}[\Phi]$ have Z_2 parities -1 and $+1$, respectively. Then, in the cubic term

$$[\Phi\Phi\Phi] = [\Phi^{(8)}\Phi^{(8)}\Phi^{(8)}] + [\Phi^{(8)}\Phi^{(8)}][\Phi] + \frac{1}{3}[\Phi^{(8)}][\Phi]^2 + \frac{1}{9}[\Phi]^3, \quad (1.14)$$

the Z_2 parities of the terms $[\Phi^{(8)}\Phi^{(8)}\Phi^{(8)}]$ and $[\Phi^{(8)}][\Phi]^2$ are -1 , so that those terms are dropped under the Z_2 symmetry:

$$[\Phi\Phi\Phi]_{Z_2=+1} = [\Phi^{(8)}\Phi^{(8)}][\Phi] + \frac{1}{9}[\Phi]^3 = [\Phi][\Phi\Phi] - \frac{2}{9}[\Phi]^3. \quad (1.15)$$

Then, by requiring a SUSY vacuum condition

$$\frac{\partial W}{\partial \Phi} = 2(\mu + \lambda[\Phi])\Phi + \lambda \left([\Phi\Phi] - \frac{2}{3}[\Phi]^2 \right) [\Phi] = 0, \quad (1.16)$$

where we have used $[\Phi^{(8)}\Phi^{(8)}] = [\Phi\Phi] - \frac{1}{3}[\Phi]^2$, we can obtain the relation (1.10). However, such the Z_2 charge assignment requires a somewhat intricate scenario [5] when Φ is related to Y , because we need not only $\Phi^{(8)}\Phi^{(8)} + \Phi^{(1)}\Phi^{(1)}$ with $Z_2 = +1$, but also $\Phi^{(8)}\Phi^{(1)} + \Phi^{(1)}\Phi^{(8)}$ with $Z_2 = -1$ in $Y = k\Phi\Phi$.

If we accept a higher dimensional term in the superpotential, by assuming a simple form without such Z_2 symmetry

$$W = \mu[\Phi\Phi] + \frac{1}{\Lambda}[\Phi]^2[\Phi^{(8)}\Phi^{(8)}], \quad (1.17)$$

we can also obtain the relation (1.10):

$$\frac{\partial W}{\partial \Phi} = 2 \left(\mu + \frac{1}{\Lambda}[\Phi]^2 \right) \Phi + \frac{2}{\Lambda} \left([\Phi\Phi] - \frac{2}{3}[\Phi]^2 \mathbf{1} \right) [\Phi] = 0. \quad (1.18)$$

However, we must recall that each Yukawaon Y_f has a different $U(1)_X$ charge $Q_X = x_f$ in order to distinguish each fermion partner [6]. Since the ur-Yukawaon Φ_e also has a $U(1)_X$ charge $Q_X = \frac{1}{2}x_e$, we cannot write the superpotential (1.17) [also Eq.(1.13)] without violating the $U(1)_X$ symmetry.

We would like to search for a superpotential form whose vacuum conditions lead to the relation (1.10) under the conditions that (i) the superpotential W does not include a higher dimensional term, and (ii) W is invariant under the $U(3)$ [or $O(3)$] and $U(1)_X$ symmetries. Note that, in the original idea (1.6), the result (1.10) is obtained independently of the explicit parameter values μ , λ and λ' . We consider that such a motive should be inherited in a SUSY version of the scenario, too. The result (1.10) should be obtained without adjusting parameters in the model. We will search for a superpotential form by considering that the form may include an ad hoc term for the time being, but the form should be simple.

2 Ansatz and VEV relations

In the present paper, we assume the Yukawaons Y_f are nonets of a $U(3)$ -flavor symmetry [or $\mathbf{5} + \mathbf{1}$ of $O(3)$], and those do not solely appear as octets of $U(3)$ [or $\mathbf{5}$ -plets of $O(3)$] in the superpotential. On the other hand, as suggested by the forms (1.11) and (1.17), the traceless part of Φ_e , $\hat{\Phi}_e \equiv \Phi_e - \frac{1}{3}[\Phi_e]$, seems to play an crucial role in obtaining the relation (1.10). Therefore, for the ur-Yukawaon Φ_e , we consider that the traceless part $\hat{\Phi}_e$ of the ur-Yukawaon can solely appear in the superpotential.

In order to obtain a bilinear relation

$$Y_e = k\Phi_e\Phi_e, \quad (2.1)$$

we assume a superpotential term [6]

$$W_A = \lambda_A[\Phi_e\Phi_e A_e] + \mu_A[Y_e A_e], \quad (2.2)$$

where $k = -\lambda_A/\mu_A$ and these fields have $U(1)_X$ charges as $Q_X(Y_e) = x_e$, $Q_X(\Phi_e) = \frac{1}{2}x_e$ and $Q_X(A_e) = -x_e$. In addition to the field A_e , we introduce a new field A'_e which couples only to $\hat{\Phi}_e$ as $[\hat{\Phi}_e\hat{\Phi}_e A'_e]$, and we also introduce a field Y'_e which composes a mass term $\mu''[Y'_e A'_e]$ together with A'_e similarly to Eq.(2.2). Since the new field A'_e has the same $U(1)_X$ charge with A_e , we can write the superpotential as follows:

$$\begin{aligned} W_A = & \lambda_A[\Phi_e\Phi_e A_e] + \mu_A[Y_e A_e] + \lambda'_A[\Phi_e\Phi_e A'_e] + \mu'_A[Y_e A'_e] \\ & + \lambda''_A[\hat{\Phi}_e\hat{\Phi}_e A'_e] + \lambda'''_A\phi_x[Y'_e A'_e], \end{aligned} \quad (2.3)$$

where $Q_X(A'_e) = -x_e$, $Q_X(\phi_x) = x_\phi$ and $Q_X(Y'_e) = x_e - x_\phi$. Here, the reason that we have written $\lambda'''_A\phi_x$ instead of μ''_A in Eq.(2.3) is to distinguish Y'_e from Y_e in order to prevent $(Y'_e)_{ij}$ from coupling with $\ell_i e_j^c$. In general, when fields A_1 and A_2 with the same $U(1)_X$ charges couple with four terms $\Phi_e\Phi_e$, Y_e , $\hat{\Phi}_e\hat{\Phi}_e$ and Y'_e in Eq.(2.3), one of those, for example, $[\hat{\Phi}_e\hat{\Phi}_e(c_1 A_1 + c_2 A_2)]$, can be rewritten as $\sqrt{c_1^2 + c_2^2}[\hat{\Phi}_e\hat{\Phi}_e A'_e]$ without losing generality. Therefore, the λ''_A -term in Eq.(2.3) is not an ansatz. However, the 6th term (λ'''_A -term) in Eq.(2.3) is, in general, given by a linear combination of A_e and A'_e . Nevertheless, we have defined Y'_e as the field Y'_e can make a mass term only with A'_e . This is just an ansatz in the present scenario. From the SUSY vacuum conditions $\partial W/\partial A_e = 0$ and $\partial W/\partial A'_e = 0$, we obtain the VEV relations (2.1) with $k = -\lambda_A/\mu_A$ and

$$Y'_e = -\frac{1}{\lambda'''_A\phi_x} \left(\lambda'_A\Phi_e\Phi_e + \lambda''_A\hat{\Phi}_e\hat{\Phi}_e + \mu'_A Y_e \right), \quad (2.4)$$

respectively. By substituting Eq.(2.1) for (2.4), we obtain a VEV relation

$$Y'_e = k'(\Phi_e \Phi_e + \xi \hat{\Phi}_e \hat{\Phi}_e), \quad (2.5)$$

where

$$k' = -\frac{1}{\lambda_A''' \phi_x} \left(\lambda'_A - \frac{\mu'_A}{\mu_A} \lambda_A \right), \quad \xi = \frac{\lambda_A''}{\lambda'_A - \frac{\mu'_A}{\mu_A} \lambda_A}. \quad (2.6)$$

(The other SUSY vacuum conditions $\partial W/\partial Y_e = 0$, $\partial W/\partial Y'_e = 0$, $\partial W/\partial \phi_x = 0$ and $\partial W/\partial \Phi_e = 0$ lead to $A_e = A'_e = 0$ for $\phi_x \neq 0$.)

Next, we introduce a field B_e with $Q_X = -\frac{3}{2}x_e + x_\phi$, and we write a superpotential term

$$W_B = \lambda_B [\Phi_e Y'_e B_e]. \quad (2.7)$$

The SUSY vacuum condition $\partial W/\partial B_e = 0$ ($W = W_A + W_B$) gives $\Phi_e Y'_e = 0$, i.e.

$$\Phi_e(\Phi_e \Phi_e + \xi \hat{\Phi}_e \hat{\Phi}_e) = (1 + \xi) \Phi_e^3 - \frac{2}{3} \xi [\Phi_e] \Phi_e^2 + \frac{1}{9} \xi [\Phi_e]^2 \Phi_e = 0, \quad (2.8)$$

from Eq.(2.5). On the other hand, in general, in a cubic equation

$$\Phi^3 + c_2 \Phi^2 + c_1 \Phi + c_0 \mathbf{1} = 0, \quad (2.9)$$

the coefficients c_i have the following relations:

$$c_2 = -[\Phi], \quad c_1 = \frac{1}{2} ([\Phi]^2 - [\Phi \Phi]), \quad c_0 = -\det \Phi. \quad (2.10)$$

The relation for the coefficient c_2

$$c_2 = -\frac{2}{3} \frac{\xi}{1 + \xi} [\Phi_e] = -[\Phi_e], \quad (2.11)$$

leads to the condition

$$(\xi + 3)[\Phi_e] = 0. \quad (2.12)$$

Considering the observed mass spectrum of the charged lepton masses, we do not choose a case of $[\Phi_e] = v_1 + v_2 + v_3 = 0$. Then the parameter ξ is required as

$$\xi = -3. \quad (2.13)$$

[If we take $\xi = -3 + \varepsilon$ ($\varepsilon \neq 0$), the model will lead to a wrong result $v_1 + v_2 + v_3 = 0$. The value of ξ must exactly be $\xi = -3$.] The constraint (2.13) puts a strong constraint on the coefficients λ_A , μ_A , λ'_A , \dots , given in the model (2.3). Since the constraint (2.13) has been settled by a physical requirement that the nonzero VEV $[\Phi_e] = v_1 + v_2 + v_3 \neq 0$ should exist, the parameter ξ is not an adjustable parameter in the phenomenological meaning.

From the relation for the coefficient c_1 , we obtain the ratio R_e defined by Eq.(1.5) as follows:
from the coefficient c_1 , we have a relation

$$c_1 = \frac{\xi}{9(1+\xi)}[\Phi_e]^2 = \frac{1}{2}([\Phi_e]^2 - [\Phi_e\Phi_e]), \quad (2.14)$$

so that we can obtain the ratio

$$R_e \equiv \frac{[\Phi_e\Phi_e]}{[\Phi_e]^2} = 1 - \frac{2\xi}{9(1+\xi)} = \frac{2}{3}, \quad (2.15)$$

by using Eq.(2.13).

Although the present model can give a reasonable value of R_e , the cubic equation (2.8) gives $c_0 = -\det\Phi_e = 0$, which means that the electron is massless, $m_e = 0$. Therefore, next, we are interested in the following ratio [7]

$$r_{123} = \frac{\sqrt{m_e m_\mu m_\tau}}{(\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau})^3} = \frac{\det\Phi_e}{[\Phi_e]^3}, \quad (2.16)$$

whose limit $r_{123} \rightarrow 1$ means that the electron is massless. A simple way to obtain a nonvanishing c_0 without affecting the values of c_1 and c_2 in the above scenario is to add an ad hoc term

$$W_{C1} = \varepsilon_1 \lambda_B [\Phi_e] [Y'_e] [B_e], \quad (2.17)$$

to the term (2.7) without violating the $U(1)_X$ symmetry. Then, the coefficient c_0 is given by

$$c_0 = \frac{\varepsilon_1}{1+\xi} [\Phi_e] \left((1+\xi) [\Phi_e\Phi_e] - \frac{\xi}{3} [\Phi_e]^2 \right). \quad (2.18)$$

By using the relations (2.13) and (2.15), we obtain

$$c_0 = \frac{1}{6} \varepsilon_1 [\Phi_e]^3, \quad (2.19)$$

so that we can obtain the ratio

$$r_{123} = -\frac{1}{6} \varepsilon_1, \quad (2.20)$$

by recalling the relation $c_0 = -\det\Phi_e$. If we consider another term

$$W_{C2} = \varepsilon_2 \lambda_B [\Phi_e Y'_e] [B_e], \quad (2.21)$$

we can also obtain

$$c_0 = 3\varepsilon_2 \det\Phi_e, \quad (2.22)$$

where we have used a formula

$$\det A = \frac{1}{3} [A^3] - \frac{1}{2} [A] [A^2] + \frac{1}{6} [A]^3. \quad (2.23)$$

Therefore, when we consider both terms (2.17) and (2.21), $W_C = W_{C1} + W_{C2}$, we obtain

$$r_{123} = -\frac{\varepsilon_1}{6(1+3\varepsilon_2)}. \quad (2.24)$$

If we assume a traceless field $\hat{B}_e \equiv B_e - \frac{1}{3}[B_e]$ instead of B_e in Eq.(2.7), the case corresponds to the case with $\varepsilon_1 = 0$ and $\varepsilon_2 = -1/3$, and we find that c_0 identically becomes $c_0 = -\det\Phi_e$, so that any value of $\det\Phi_e$ is allowed. Therefore, the case is not so interesting. At present, the parameters ε_1 and ε_2 are free, so that we cannot predict the value of r_{123} .

3 Concluding remarks

In conclusion, we have found a superpotential which can lead to the VEV relation (1.10), $[\Phi_e\Phi_e] = \frac{2}{3}[\Phi_e]^2$. It should be noticed that, although we have assumed a U(3) [or O(3)] flavor symmetry in the present paper, it does not mean that the relation (1.10) was derived by assuming the symmetry. The relation (1.10) was obtained by assuming a specific form (2.3) in the superpotential under the flavor symmetry. In the superpotential (2.3), the existence of the term $\Phi_e\hat{\Phi}_e\hat{\Phi}_e$ plays an crucial role in obtaining the relation (1.10). If all allowed terms under the symmetry were indiscriminately taken into consideration, the model would have become a “parameter physics” as well as conventional mass matrix models. [We have chosen ξ as $\xi = -3$ in Eq.(2.13). However, as discussed in the previous section (below Eq.(2.13)), we do not regard ξ as an adjustable parameter in the present model.]

Since we have successfully obtained the relation (1.10) without adjustable parameters, another problem has risen in the present scenario: We know that $R = 2/3$ is valid only for the charged lepton masses, and the observed masses for another sectors do not satisfy $R = 2/3$. For example, the ratio R_u for the up-quark masses is $R_u \simeq 8/9$ [8]. Can we modify the present scenario as it leads to $R_u \simeq 8/9$? At present it seems to be impossible, because there is no adjustable parameter in the present scenario.

By the way, on the basis of a Yukawaon model, an interesting neutrino mass matrix form [6, 9]

$$M_\nu \propto \langle Y_e \rangle (\langle Y_e \rangle \langle \Phi_u \rangle + \langle \Phi_u \rangle \langle Y_e \rangle)^{-1} \langle Y_e \rangle, \quad (3.1)$$

has been proposed, where the up-quark mass spectrum is given by $\langle Y_u \rangle \propto \langle \Phi_u \rangle \langle \Phi_u \rangle$. The neutrino mass matrix (3.1) can successfully lead to a nearly tribimaximal mixing [11] under an additional phenomenological assumption. In the successful description of M_ν , it is crucial that the Majorana mass matrix of the right-handed neutrinos M_R is given by linear terms of $\sqrt{m_{ui}}$. Therefore, the bilinear form $\langle Y_u \rangle \propto \langle \Phi_u \rangle \langle \Phi_u \rangle$ seems to be valid for the up-quark sector, too.

We also pay attention to the following empirical relation

$$\sqrt{\frac{m_{ui}}{m_{uj}}} \simeq \frac{m_{ei} + m_0}{m_{ej} + m_0}, \quad (3.2)$$

where m_{ui} and m_{ei} are masses of up-quarks and charged leptons. In fact, for example, the value $m_0 = 4.36$ MeV gives the ratios $(m_e + m_0)/(m_\mu + m_0) = 0.0453$ and $(m_\mu + m_0)/(m_\tau + m_0) = 0.0612$ correspondingly to the observed values $\sqrt{m_u/m_c} = 0.0453^{+0.012}_{-0.010}$ and $\sqrt{m_c/m_t} =$

$0.0600^{+0.0045}_{-0.0047}$, respectively. (Here, we have used quark mass values [10] at $\mu = m_Z$, because the quark mass values at a unification scale are highly dependent on the value of $\tan \beta = v_u/v_d$.)

These facts suggest a possibility that

$$\langle Y_e \rangle \propto \langle \Phi_e \rangle \langle \Phi_e \rangle, \quad \langle \Phi_e \rangle \equiv \langle \Phi_0^e \rangle, \quad (3.3)$$

in the charged lepton sector, while

$$\langle Y_u \rangle \propto \langle \Phi_u \rangle \langle \Phi_u \rangle, \quad \langle \Phi_u \rangle \propto \langle \Phi_0^u \rangle \langle \Phi_0^u \rangle + \varepsilon \mathbf{1}, \quad (3.4)$$

in the up-quark sector, where ur-Yukawaons Φ_0^e and Φ_0^u exactly have the same VEV spectra, but the diagonal bases of $\langle \Phi_0^e \rangle$ and $\langle \Phi_0^u \rangle$ are different from each other.

Therefore, there is a possibility that all quark and lepton mass spectra (in other words, all $\langle Y_f \rangle$) can be described in terms of only two ur-Yukawaons Φ_0^e and Φ_0^u . However, in Ref.[6, 1], where a supersymmetric Yukawaon model has been investigated on the basis of an O(3) flavor symmetry, the down-quark Yukawaon Y_d has not explicitly been discussed. In the O(3) model [6, 1], since it is assumed that the VEVs of Φ_0^e and Φ_0^u are real, the observed CP violating phase in the quark sector must be inevitably included in the down-quark sector. Whether such a unified description is possible or not is dependent on whether a down-quark Yukawaon Y_d can also reasonably be described in terms of Φ_0^e and Φ_0^u . This will be a touchstone of the Yukawaon approach.

Finally, we would like to comment on soft SUSY breaking terms. All the results in the present paper have been derived from using the SUSY vacuum conditions, $\partial W / \partial A_e = 0$, and so on. We know that SUSY is broken in the realistic world. If we add a soft SUSY breaking term, our results will be changed in principle. Since we consider that all mass parameters in the superpotential (2.3) and the VEVs of Y_e and Φ_e are of the order of $\Lambda \sim 10^{15}$ GeV (Λ is an energy scale of the effective theory, and the value $\Lambda \sim 10^{15}$ GeV is estimated from a Yukawaon model [6] for the neutrino sector), the effects will be negligibly small, i.e. $m_{soft}/\Lambda \sim 10^3 \text{ GeV} / 10^{15} \text{ GeV} \sim 10^{-12}$. However, the effect for the conditions (2.13) is troublesome. In order to keep $\xi = -3$ exactly, we must assume that such a soft symmetry breaking term which contribute to Eq.(2.13) is exactly zero. As we have stated in Sec.2, we consider that the constraint $\xi = -3$ is not a phenomenological one, but a fundamental one in the model, although, at present, we do not know such a reasonable mechanism which keeps $\xi = -3$. (Or, we may assume that SUSY is unbroken as far as Yukawaons are concerned.) More details for possible soft breaking terms will be discussed elsewhere.

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